IV. Summary

The capacitance of a rectangular metal line over a conducting ground plane, for $W/h \gtrsim 1$, is

$$C = 2\varepsilon/\pi \ln 2R_b/R_a$$

$$\ln R_a = -1 - \pi W/2h - [(p+1)/p^{1/2}] \tanh^{-1} p^{-1/2} - \ln [(p-1)/4p]$$

$$R_b = \eta + [(p+1)/2p^{1/2}] \ln \Delta$$

$$\eta = p^{1/2}\{\pi W/2h + [(p+1)/2p^{1/2}][1 + \ln 4/(p-1)] - 2 \tanh^{-1} p^{-1/2}\}$$

$$\Delta = \text{larger value, } \eta \text{ or } p$$

$$p = 2B^2 - 1 + \sqrt{(2B^2-1)^2 - 1}$$

$$B = 1 + t/h. \qquad (24)$$

The accuracy of (24) is about 1 percent when $W/h \gtrsim 1$.

The capacitance of a rectangular metal line with two conducting ground planes, for $W/h \geq 0.5$, $d/h \geq 0.5$, is

$$C/\varepsilon = 2/\pi \ln R_B/R_A$$

$$\ln R_A = -\pi W/2h - 2\alpha \tanh^{-1} \sqrt{(p+q)/p(1+q)} + 2\gamma \tanh^{-1} p^{-1/2} + \ln 4p/(p-1)$$

$$\ln R_B = \gamma^{-1}\{\pi W/2h + 2\alpha \tanh^{-1} \sqrt{(1+q)/(p+q)} + \gamma \ln (p-1)/4 - 2 \tanh^{-1} p^{-1/2}\}$$

$$\alpha = (h + d + t)/h$$

$$\gamma = d/h$$

$$p = q^2/\gamma^2$$

$$q = \tfrac{1}{2}[\alpha^2 - \gamma^2 - 1 + \sqrt{(\alpha^2 - \gamma^2 - 1)^2 - 4\gamma^2}]. \qquad (25)$$

The errors are less than 1 percent. Both formulas are more accurate for a wider and/or a thicker metal line. Once the capacitance is found, the characteristic impedance of the composite transmission line is easily obtained. Aside from a constant, the characteristic impedance is the inverse of the capacitance.

References

[1] H. B. Palmer, "Capacitance of a parrel-plate capacitor by the Schwartz-Christoffel transformation," *Trans. AIEE*, vol. 56, p. 363, March 1937.
[2] F. Assadourian and E. Rimai, "Simplified theory of microstrip transmission systems," *Proc. IRE*, p. 1651, Dec. 1952.
[3] H. A. Wheeler, "Transmission-line properties of parallel wide strips by a conformal-mapping approximation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, p. 280, May 1964.
[4] S. B. Cohn, "Characteristic impedance of the shielded strip transmission line," *IRE Trans. Microwave Theory Tech.*, vol. MTT-2, p. 52, July 1954.
[5] H. Guckel, "Characteristic impedances of generalized rectangular transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, p. 270, May 1965.
[6] J. K. Richardson, "An approximate formula for calculating Z_0 of a symmetric strip line," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-15, p. 130, Feb. 1967.
[7] W. T. Weeks, "Calculation of coefficients of capacitance for multiconductor transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, p. 35, Jan. 1970.
[8] M. V. Schneider, "Microstrip lines for microwave integrated circuits," *Bell Sys. Tech. J.*, vol. 48, p. 1421, May–June 1969.

Equivalent Circuit of a Narrow Axial Strip in Waveguide

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Abstract—**A theoretical determination is made of the two-port equivalent circuit of a narrow strip located axially in a rectangular waveguide such that it extends partially or completely across the waveguide narrow dimension. The analysis is based upon derivation of a variational expression for a field quantity from which can be determined the reflection coefficient and the equivalent-circuit parameters. Experimental input susceptance values agree closely with the theory. The analysis shows that the T-equivalent network of a nontouching strip has a series-resonant shunt circuit. This element has application in filter and impedance-transforming networks, in planar circuits, and in fin-line structures.**

I. Introduction

This short paper presents a theoretical determination, with experimental verification, of the equivalent circuit of a narrow infinitesimally thin perfectly conducting strip which is partially or completely inserted in a rectangular waveguide in such a manner that the strip surface is parallel to the narrow waveguide wall.

Konishi *et al.* [1]–[3] have developed a method for the design of planar circuits, by which the circuit elements are located on a metal sheet which is inserted axially into a waveguide; its advantages include low cost and ease of mass production. Meier [4]–[7] has advocated fin line, in which metal fins printed on a dielectric substrate bridge the broad walls of a rectangular waveguide, as a propagating structure for millimeter-wave integrated circuits. The geometry considered in this short paper belongs to the general form defined by those papers. Although the analysis presented here is restricted to narrow strips, it is applicable to the design of bandpass filters, diode mounts, and tuning elements of the form described by Konishi *et al.* [2], [3].

The narrow axial nontouching thin strip has not previously been subjected to theoretical analysis. Konishi *et al.* [2], [3] used a Rayleigh–Ritz variational technique to obtain an equivalent circuit for a uniform strip which extends across the entire waveguide height. Their method requires knowledge of the modes in the two sections of the waveguide bifurcated by the strip; thus it is not readily applicable to the nontouching strip.

The approach used here is based upon the variational method used previously by the present authors for the analysis of a thin

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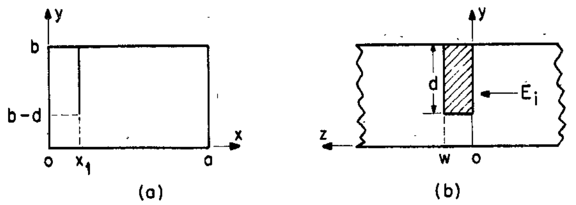

Fig. 1. The narrow axial strip in rectangular waveguide.

transverse nontouching strip in guide [8]. However, further development is necessary to apply that theory to the axial strip because of the greater difficulty in obtaining a variational form, and also because the axial strip must be represented by a two-port network.

II. Reflection-Coefficient Derivation

The structure being analyzed here is shown in Fig. 1 for the case where the strip does not extend across the entire waveguide height; the special case of a touching strip is readily treated by this analysis, putting $d = b$. The strip is assumed to be perfectly conducting (as is the waveguide) and to be infinitesimally thin. The strip width w is assumed to be small compared to the dominant-mode waveguide wavelength, so that the current does not vary appreciably with z in the range $0 \leq z \leq w$; this restriction may be removed through a generalization of the analytical approach presented here.

Consider a dominant-mode incident electric field, we assume that the scattered field lies only in the $\hat{y}$ direction, and the Green's function $\tilde{G}(r \mid r')$ takes the same form as in [8] with the exception that $\exp(-\Gamma_{nm}|z|)$ is replaced by $\exp(-\Gamma_{nm}|z - z'|)$.

The total field at any point r vanishes on the surface S

$$\sin\left(\frac{\pi x_1}{a}\right) \exp(-\Gamma_{10} z) - j\omega\mu_0 \int_S [G_y(r \mid r')]_{x=x_1} \cdot J_y(y',z')\, dy'\, dz' = 0 \quad \text{on} \quad S. \qquad (1)$$

For a narrow strip, $J_y(y',z')$ will not vary appreciably[1] with z' in the range $0 \leq z' \leq w$, the reflection coefficient R for the dominant mode at $z = 0$ may be found as

$$R = \left(\frac{-j\omega\mu_0}{ab\Gamma_{10}}\right) \sin\left(\frac{\pi x_1}{a}\right) \left\{-\frac{1}{\Gamma_{10}} [\exp(-\Gamma_{10} w) - 1]\right\} \cdot \int_{b-d}^{b} J_y(y')\, dy'. \qquad (2)$$

In [8] the R expression could be directly substituted in the E_t expression; a more involved derivation is necessary here.

Equation (1) can be manipulated into the form

$$\begin{aligned} &\sin\left(\frac{\pi x_1}{a}\right) \left\{\frac{1}{\Gamma_{10}} [1 - \exp(-\Gamma_{10} w)]\right\} \int_{b-d}^{b} J_y(y)\, dy \\ &- \frac{j\omega\mu_0}{ab\Gamma_{10}} \sin^2\left(\frac{\pi x_1}{a}\right) \left\{\frac{2w}{\Gamma_{10}} + \frac{2}{\Gamma_{10}^2} [\exp(-\Gamma_{10} w) - 1]\right\} \\ &\cdot \int_{b-d}^{b} \int_{b-d}^{b} J_y(y) J_y(y')\, dy'\, dy \\ &= j\omega\mu_0 \left(\sum_{n=2}^{\infty} \sum_{m=0}^{\infty} + \sum_{\substack{m=1; \\ \text{at } n=1}}^{\infty}\right) \frac{(2 - \delta_m)\left(k_0^2 - \frac{m^2\pi^2}{b^2}\right)}{abk_0^2 \Gamma_{nm}} \\ &\cdot \sin^2\left(\frac{n\pi x_1}{a}\right) \left\{\frac{2w}{\Gamma_{nm}} + \frac{2}{\Gamma_{nm}^2} [\exp(-\Gamma_{nm} w) - 1]\right\} \\ &\cdot \int_{b-d}^{b} \int_{b-d}^{b} J_y(y) J_y(y') \cos\left(\frac{m\pi y}{b}\right) \\ &\cdot \cos\left(\frac{m\pi y'}{b}\right) dy'\, dy \end{aligned} \qquad (3)$$

through multiplication by $J_y(y)$ and integration with respect to y and z over the strip surface at $x = x_1$. Note that this equation applies to points located on the strip, i.e., for $0 \leq z \leq w$.

The left-hand side of this equation may be rewritten, using (2), as

$$(A + HR) \sin\left(\frac{\pi x_1}{a}\right) \int_{b-d}^{b} J_y(y)\, dy$$

where

$$A = \frac{1}{\Gamma_{10}} [1 - \exp(-\Gamma_{10} w)]$$

$$H = \frac{2w + \frac{2}{\Gamma_{10}} [\exp(-\Gamma_{10} w) - 1]}{1 - \exp(-\Gamma_{10} w)}.$$

Using this form, together with (2) at the $z = 0$ plane where both equations hold, we can form an expression $L = -2R/(A + HR)$ given by

$$L = \frac{\left(\frac{2}{\Gamma_{10}^2}\right) \sin^2\left(\frac{\pi x_1}{a}\right) [1 - \exp(-\Gamma_{10} w)] \int_{b-d}^{b} \int_{b-d}^{b} J_y(y) J_y(y')\, dy'\, dy}{\left(\sum_{n=2}^{\infty} \sum_{m=0}^{\infty} + \sum_{\substack{m=1; \\ \text{at } n=1}}^{\infty}\right) \frac{(2 - \delta_m)\left(k_0^2 - \frac{m^2\pi^2}{b^2}\right)}{k_0^2 \Gamma_{nm}} \left\{\frac{2w}{\Gamma_{nm}} + \frac{2}{\Gamma_{nm}^2} [\exp(-\Gamma_{nm} w) - 1]\right\} \sin^2\left(\frac{n\pi x_1}{a}\right) \cdot \int_{b-d}^{b} \int_{b-d}^{b} J_y(y) J_y(y') \cos\left(\frac{m\pi y}{b}\right) \cos\left(\frac{m\pi y'}{b}\right) dy'\, dy}. \qquad (4)$$

Using a method similar to Lewin [9], this expression for L is readily shown to be stationary for small variations in $J_y(y)$ about its correct value. The definition of L compares with the susceptance form $-2R/(1 + R)$ found to be stationary for the transverse strip [8]. The additional complexity here arises from the two-port nature of the axial strip impedance.

Using an approximate form for $J_y(y)$, we can now determine L and hence $R = -AL/(HL + 2)$.

III. Reflection-Coefficient Evaluation

For the nontouching strip, the L expression in (4) can be evaluated using an approximate current distribution determined

[1] Our analysis initially used: $J(y',z') = J(y') \exp(-\Gamma_{10} z')$ rather than $J(y',z') = J(y')$. However, for strip widths of interest, the susceptance values obtained were almost exactly equal to those obtained for the simpler current form. We have consequently preferred this form, due to the relative simplicity of the resulting algebra.

from the analysis of [8] to be in the form

$$J_y(y) = \sin k_1(y - b + d)$$

where k_1 may be equal to either k_0 or $\pi/2d$ without significant effect on the resulting value, for operation above the waveguide dominant-mode cutoff frequency.

Using this distribution we obtain

$$L = \frac{\left(\frac{2}{\Gamma_{10}^2}\right)(1 - \cos k_1 d)^2[1 - \exp(-\Gamma_{10}w)] \sin^2\left(\frac{\pi x_1}{a}\right)}{\sum_{n=2}^{\infty} C_n + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B_{nm}} \tag{5}$$

where

$$C_n = \left(\frac{1}{\Gamma_{n0}}\right)(1 - \cos k_1 d)^2 \left\{\frac{2w}{\Gamma_{n0}} + \frac{2}{\Gamma_{n0}^2} \cdot [\exp(-\Gamma_{n0}w) - 1]\right\} \sin^2\left(\frac{n\pi x_1}{a}\right)$$

and

$$B_{nm} = \left[\frac{2\left\{k_0^2 - \left(\frac{m^2\pi^2}{b^2}\right)\right\}}{k_0^2\Gamma_{nm}}\right] \cdot \left\{\frac{2w}{\Gamma_{nm}} + \frac{2}{\Gamma_{nm}^2}[\exp(-\Gamma_{nm}w) - 1]\right\}\left[\sin^2\left(\frac{n\pi x_1}{a}\right)\right] \cdot \left\{\frac{k_1^4\left[\cos\frac{m\pi}{b}(b - d) - \cos(m\pi + k_1 d)\right]^2}{\left[k_1^2 - \left(\frac{m^2\pi^2}{b^2}\right)\right]^2}\right\}.$$

For the touching strip, substitution of a constant current in (4) yields

$$L = \frac{\left(\frac{2}{\Gamma_{10}^2}\right)[1 - \exp(-\Gamma_{10}w)] \sin^2\left(\frac{\pi x_1}{a}\right)}{\sum_{n=2}^{\infty} D_n} \tag{6}$$

where

$$D_n = \frac{2}{\Gamma_{n0}^2}\left\{w + \frac{1}{\Gamma_{n0}}[\exp(-\Gamma_{n0}w) - 1]\right\} \sin^2\left(\frac{n\pi x_1}{a}\right).$$

The accuracy of the expression for L in (5) and (6), which are derived using the approximate forms for current distribution, must now be assessed through experimental measurements.

IV. Experimental Verification

The axial obstacles shown in Fig. 2(a) and (b) may be represented by a T-equivalent network, as shown in Fig. 2(c); because of the longitudinal symmetry of the structure, the series arms of this circuit are equal and the calculation is simplified. When the waveguide is terminated in a match, the normalized input admittance may be represented by $\bar{Y}_{in}$, as shown in Fig. 2(c). Theoretical values of $\bar{Y}_{in}$ may be found from R, using the expression for L, and the susceptive component $\bar{B}_{in}$ compared with experimental measurements.

$$\bar{Y}_{in} = \frac{1 - R}{1 + R} = \frac{2 + (H + A)L}{2 + (H - A)L}. \tag{7}$$

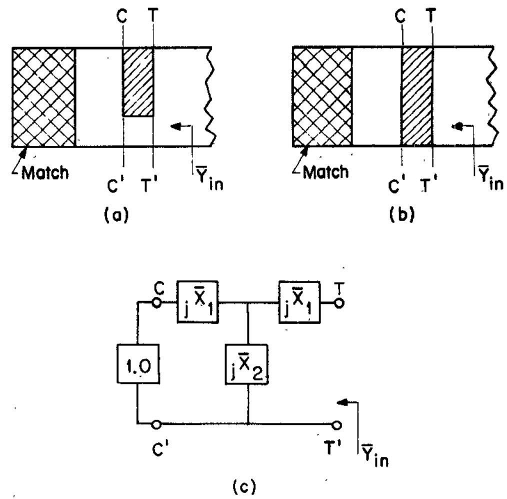

Fig. 2. The axial-strip circuit with a matched termination. (a) A nontouching strip. (b) A touching strip. (c) Two-port equivalent circuit.

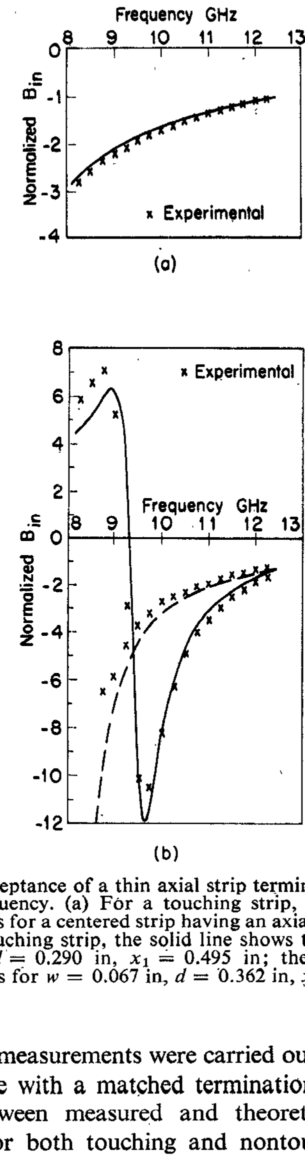

Fig. 3. Input susceptance of a thin axial strip terminated in a match as a function of frequency. (a) For a touching strip, the solid line shows theoretical results for a centered strip having an axial width $w = 0.125$ in. (b) For a nontouching strip, the solid line shows theoretical results for $w = 0.133$ in, $d = 0.290$ in, $x_1 = 0.495$ in; the broken line shows theoretical results for $w = 0.067$ in, $d = 0.362$ in, $x_1 = 0.450$ in.

Experimental measurements were carried out to determine the input admittance with a matched termination. Fig. 3 shows a comparison between measured and theoretically determined values of $\bar{B}_{in}$ for both touching and nontouching strips. The agreement is good and indicates the accuracy of the theory; similar agreement was obtained for a wide variety of strip dimensions. In Fig. 3(b), for the nontouching strip, the degree of agree-

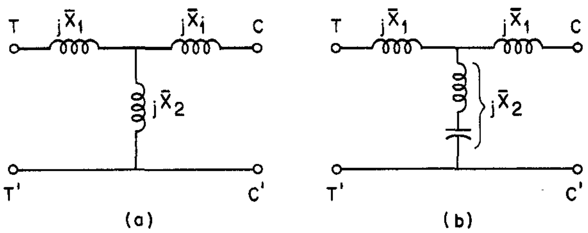

Fig. 4. Equivalent circuits for a thin axial strip. (a) For a touching strip. (b) For a nontouching strip.

ment is not quite as close as for the touching strip, due to measurement errors in determining the depth to which the strip had been inserted in the waveguide. It is evident that a resonant effect occurs for nontouching strips; this is explored further in the discussion below on equivalent-circuit element values.

V. Equivalent-Circuit Element Values

Referring to Fig. 2(c), we must determine the equivalent-circuit values $\bar{X}_1$ and $\bar{X}_2$ from the $\bar{Y}_{in}$ value found with a matched termination at the port $C - C'$. Equating the input admittance from Fig. 2(c) to $\bar{Y}_{in}$ yields these expressions for $\bar{X}_1$ and $\bar{X}_2$

$$\bar{X}_1 = -\bar{X}_2 \pm \frac{Q}{P-1} \tag{8a}$$

$$\bar{X}_2 = \pm \left\{ P \left[1 + \left(\frac{Q}{P-1} \right)^2 \right] \right\}^{1/2} \tag{8b}$$

where

$$P = \frac{\bar{G}_{in}}{\bar{G}_{in}^2 + \bar{B}_{in}^2}$$

$$Q = \frac{-\bar{B}_{in}}{\bar{G}_{in}^2 + \bar{B}_{in}^2}.$$

Substitution for P and Q in (8) gives four sets of values for $\bar{X}_1$ and $\bar{X}_2$. Two sets are rejected, since they give the conjugate value $\bar{Y}_{in}^*$. We then imposed the additional restriction that the equivalent-circuit element values satisfy the Foster reactance theorem requirement that $dX/d\omega > 0$ for a lossless element. This restricted the acceptable values to one set only.

The set of values for $\bar{X}_1$ and $\bar{X}_2$ is presented in the subsequent graphs for strips in X-band waveguide. The equivalent circuits for touching and nontouching strips take the form shown in Fig. 4, where the element values are functions of geometry and frequency.

Further confirmation of the theory is shown in Fig. 5 where experimental values of $\bar{B}_{in}$ for an axial-centered strip in a short-circuited waveguide are compared with theoretical values computed using $\bar{X}_1$ and $\bar{X}_2$.

Fig. 6 shows the variation in $\bar{X}_1$ and $\bar{X}_2$ with width for a centered strip. The general features of Fig. 6(a) compare favorably with a similar curve computed by Konishi *et al.* [3] using a Rayleigh–Ritz approach. It should be noted that as the width increases, the assumption that the current distribution is independent of z will require closer examination, and the theoretical expression modified.

For a nontouching strip, the effect of variation in depth of strip insertion into the guide is presented in Fig. 7(a), from which it is seen that $\bar{X}_1$ is insensitive to depth, while the resonant frequency of $\bar{X}_2$ is determined by this depth. Such curves obtain a plot of resonant frequency as a function of depth, for various strip widths, shown in Fig. 7(b); neglecting the dependence on strip width, the resonant frequency is approximately that for which the depth is one-quarter the free-space wavelength. A similar resonance is obtained with a transverse nontouching strip, as shown in [8, fig. 4(a)].

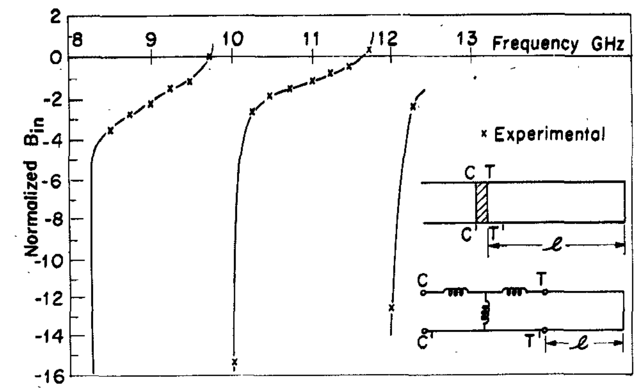

Fig. 5. Normalized input susceptance of a thin axial touching strip in a short-circuited waveguide as a function of frequency. The strip width is $w = 0.070$ in and its edge is $l = 2.298$ in from the short circuit. The solid line shows theoretical values.

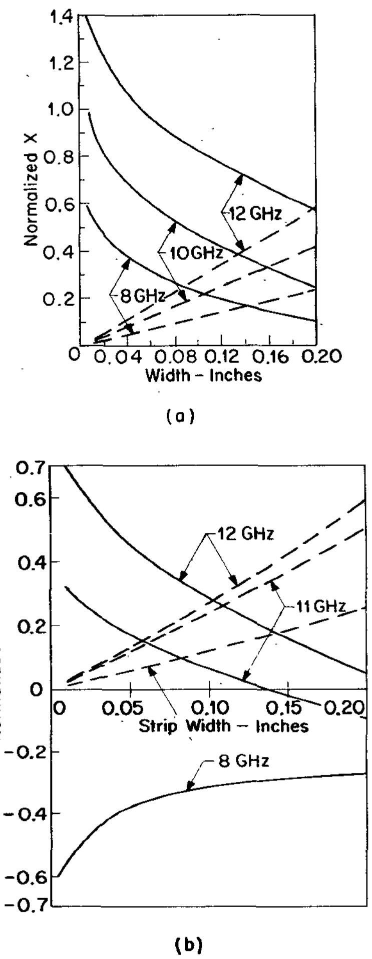

Fig. 6. Equivalent circuit element values $\bar{X}_1$ and $\bar{X}_2$ as a function of strip width. The solid line shows $\bar{X}_2$ and the broken line shows $\bar{X}_1$. (a) For a centered touching strip. (b) For a centered nontouching strip having $d = 0.300$ in.

VI. Conclusions

A variational expression has been found here, which can be used to determine the reflection coefficient and hence the equivalent-circuit element values for a narrow axial strip in

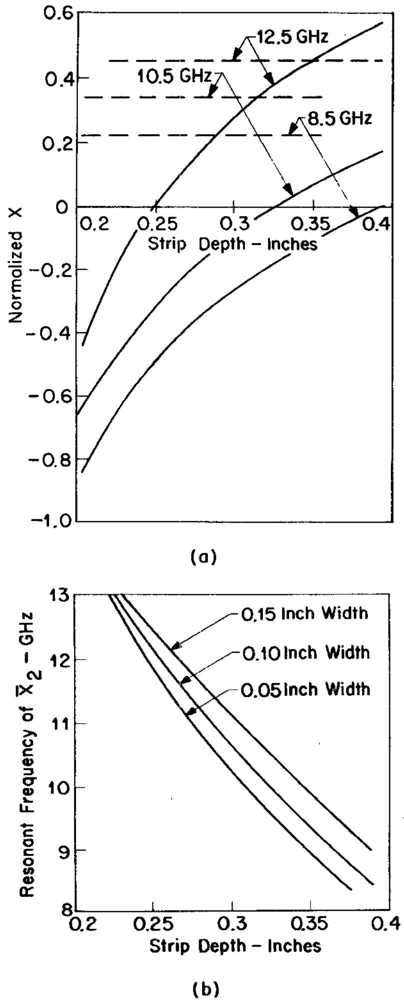

Fig. 7. The effect of insertion depth of a nontouching strip on the equivalent-circuit element values. (a) For a centered strip having $w = 0.15$ in; the solid line shows X_2 and the broken line shows X_1. (b) The resonant frequency of X_2 as a function of depth for various values of w.

rectangular waveguide. The experimental values of input susceptance agree closely with the theoretical values. The resulting equivalent circuit has direct application in the design of microwave filters and tuning elements, and in the recently proposed planar circuits and fin-line structures.

References

[1] Y. Konishi, K. Uenakada, N. Yazawa, N. Hoshino, and T. Takahashi, "Simplified 12-GHz low-noise converter with mounted planar circuit in waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 451–454, April 1974.
[2] Y. Konishi, K. Uenakada, and N. Hoshino, "The design of planar circuit mounted in waveguide and the application to low noise 12-GHz converter," *1974 IEEE S-MTT International Microwave Symposium Digest*, pp. 168–170.
[3] Y. Konishi and K. Uenakada, "The design of a bandpass filter with inductive strip—planar circuit mounted in waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 869–873, October 1974.
[4] P. J. Meier, "Two new integrated-circuit media with special advantages at millimeter wavelengths," *1972 IEEE-GMTT International Microwave Symposium Digest*, pp. 221–223.
[5] ——, "Equivalent relative permittivity and unloaded Q factor of integrated finline," *Electronics Letters*, vol. 9, pp. 162–163, April 5, 1973.
[6] ——, "Integrated fin-line millimeter components," *1974 IEEE S-MTT International Microwave Symposium Digest*, pp. 195–197.
[7] ——, "Integrated fin-line millimeter components," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 1209–1216, December 1974.
[8] K. Chang and P. J. Khan, "Analysis of a narrow capacitive strip in waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 536–541, May 1974.
[9] L. Lewin, *Advanced Theory of Waveguides*. London, England: Iliffe, 1951.

On the Theory of Coupling Between Finite Dielectric Resonators

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Abstract—**The coupling coefficients between open dielectric resonators in three useful dielectric-filter configurations, calculated from an electric rather than from a magnetic excitation of the fields, are given. The limitations of the latter method are pointed out and experimental results are given which supports the first method and shows that the differences cannot always be neglected.**

Introduction

Mutually coupled dielectric resonators in a waveguide under cutoff form a useful class of microwave bandpass filters [1], [2]. Since an exact field analysis of such a filter is a formidable problem, various approximate methods have been used. In order to calculate the resonance frequencies and internal fields of the resonators, one usually uses the so-called magnetic-wall model [3]. To calculate the coupling between two resonators or between a resonator and the waveguide fields various magnetic-dipole approximations have been used [1], [2], [4], [5]. In this short paper we will treat these coupling problems by calculating the excited waveguide fields directly from the polarization current density. This is also shown to be theoretically more correct than the magnetic-dipole methods. When we make the actual calculations we use the magnetic-wall model to obtain the polarization currents. We then get formulas which are almost as easy to use as the ones previously used. These formulas indicate that the commonly used magnetic-dipole approximations can give substantial errors in coupling strength. Experimental results obtained with two TiO_2-resonators, $\varepsilon_r = 90$, also support this method compared to the magnetic-dipole methods.

The Excitation Amplitude

Suppose that we in some way have found the fields inside the resonator and want to find the related waveguide fields for $z > A/2$, in Fig. 1. To do this we will use the polarization current $\bar{J}_p$ as current density $\bar{J}$ in the waveguide or, if $\varepsilon_{r2} \neq 1$, the excess polarization current density $j\omega\, \varepsilon_0(\varepsilon_{r1} - \varepsilon_{r2})\bar{E}$. In the following we will, however, assume that $\varepsilon_{r2} = 1$.

We expand the waveguide fields in orthogonal modes as

$$\bar{E}^{\pm} = \sum_n C_n^{\pm}(\bar{e}_n \pm \bar{e}_{zn})e^{\mp j\beta_n z} = \sum_n C_n^{\pm}\bar{E}_n^{\pm} \qquad (1)$$

$$\bar{H}^{\pm} = \sum_n C_n^{\pm}(\pm \bar{h}_n + \bar{h}_{zn})e^{\mp j\beta_n z} = \sum_n C_n^{\pm}\bar{H}_n^{\pm} \qquad (2)$$

and get [6]

$$C_n^{+} = -\frac{1}{P_n}\int_V \bar{J}_p \cdot \bar{E}_n^{-}\, dV \qquad (3)$$

where

$$P_n = 2\int_{\substack{\text{waveguide}\\ \text{cross}\\ \text{section}}} \bar{e}_n \times \bar{h}_n \cdot \hat{z}\, ds. \qquad (4)$$

So far we have not introduced any approximations. In the magnetic-wall model we may obtain expressions for the fields inside the resonator [3]. In this model the surfaces $y' = \pm B/2$ and $z' = \pm A/2$ in Fig. 1 are perfect magnetic conductors. It is

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